

## eSpyMath: AP Precalculus Practice Questions (2024)

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1. If  $\alpha$  and  $\beta$  are the roots of the quadratic equation of  $x^2 - 4x + 2 = 0$ , determine  $(\alpha + \beta) \times (\alpha \times \beta)$

2. If  $\alpha$  and  $\beta$  are the roots of the quadratic equation of  $x^2 - 4x + 2 = 0$ , determine  $|\alpha - \beta|$

3. Find  $f \circ g(4)$  where  $f\left(\frac{1}{x}\right) = \sqrt{x} - 2$  and  $g(x) = 2x$

4. For what value of  $a$ , the value of the term of  $ax^2 - 2x + 3a$  is always positive?

5. What is the numerical coefficient of the term containing  $u^4v^3$  in the expansion of  $(3u + v)^7$ ?

6. What is the inverse of the matrix  $\begin{bmatrix} 5 & -1 \\ 7 & 2 \end{bmatrix}$ ?

7. If  $g(x) = x^4 + Cx^3 + Dx^2 + 6$ , and  $g(2) = 8$  and  $g(-2) = -32$ , what is the value of  $3C + D$ ?

8. If  $h(x) = x^5 - 3x^3 + 2x - 4$ , find  $h(-1)$ .

9. What is the domain of  $h(x) = \frac{x^2 - 9}{x^3 - 27}$ ?

10. What is the domain of  $p(x) = \frac{\sqrt{2x - 6}}{x^2 - 4x + 4}$ ?

11. Which of the following represents the equation of  $k^{-1}(x)$  for the inverse of the function  $k(x) = 2^{x+2} - 4$ ?

12. Which of the following is equivalent to the expression  $\log_5 \left( \frac{p^4 q^3}{r^2} \right)$ ?

(A)  $\frac{4}{5} \log_5 p + \frac{3}{5} \log_5 q - \frac{2}{5} \log_5 r$

(B)  $\frac{1}{5} (4 \log_5 p + 3 \log_5 q) - \frac{2}{5} \log_5 r$

(C)  $4 \log_5 p + 3 \log_5 q - 2 \log_5 r$

(D)  $\frac{4}{2} \log_5 p + \frac{3}{2} \log_5 q - \log_5 r$

13. Identify the vertical asymptote(s) for the function  $g(x) = \frac{x^3 - 1}{x^2 - x - 6}$ .

14. Given  $h(x) = \begin{cases} -2x^2 + 8, & x \leq 1 \\ \sqrt{x+4}, & x > 1 \end{cases}$ , find  $h(0.5)$ .

15. Find the slant asymptote of  $m(x) = \frac{2x^2 - 5x}{x - 2}$ .

16. In polar coordinates, which of the following choices is not equivalent to  $(3, -\frac{\pi}{4})$ ?

(A)  $(3, \frac{7\pi}{4})$

(B)  $(-3, \frac{3\pi}{4})$

(C)  $(3, \frac{15\pi}{4})$

(D)  $(-3, -\frac{5\pi}{4})$

17. Which of the following represents zeros of  $s = 3 - 3\cos 2\theta$ ?

(A)  $\frac{\pi}{6}, \frac{5\pi}{6}$

(B)  $0, \frac{\pi}{2}, \pi$

(C)  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(D)  $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

18. Which of the following choices represents the corresponding rectangular equation of the curve with the parametric equations  $x(t) = 4t$ ,  $y(t) = t^2 - 2t + 3$ ?

(A)  $y = \frac{1}{16}x^2 - \frac{1}{2}x + 3$

(B)  $y = \frac{1}{4}x^2 - 2x + 3$

(C)  $y = x^2 - 8x + 12$

(D)  $y = 4x - x^2 + 3$

19. Evaluate:  $\sin\left(\arctan\left(\frac{4}{3}\right)\right)$ , given that  $0 \leq \theta \leq \frac{\pi}{2}$ .

20. Given that  $\cos \alpha = \frac{3}{5}$  and  $\sin \alpha < 0$ , find  $\sin \alpha$ .

(A)  $\sqrt{\frac{16}{25}}$

(B)  $-\sqrt{\frac{16}{25}}$

(C)  $\frac{4}{5}$

(D)  $-\frac{4}{5}$

21. Which of the following is a possible equation for the sinusoidal graph shown with a maximum at  $y = 4$ , a minimum at  $y = -2$ , and a period of 8?

(A)  $y = 3\sin\left(\frac{\pi}{4}x\right) + 1$

(B)  $y = 3\cos\left(\frac{\pi}{4}x\right) - 1$

(C)  $y = 6\sin\left(\frac{\pi}{4}x\right) + 1$

(D)  $y = 6\cos\left(\frac{\pi}{4}x\right) + 1$

22. Which of the following is equivalent to  $2\sin(7x)\cos(2x) + 2\cos(7x)\sin(2x)$ ?

(A)  $2\sin 5x$

(B)  $2\sin 9x$

(C)  $2\cos 5x$

(D)  $2\cos 9x$

23. Given  $\cos x = -1/3$  and  $\tan x > 0$ , find  $\cos 2x$ .

24. Determine the period of the function  $y = -3\cos\left(2\left(x + \frac{\pi}{4}\right)\right)$ .

25. Solve the equation  $\log_d(5 - \log_d(m)) = n$  for  $m$ .

26. Determine whether the function  $h(x) = x^6 - x^2 \cos(x)$  is odd, even, or neither.

27. Find the linear equation for the function  $p$  that passes through the points  $(2, -5)$  and  $(-4, 3)$ .

28. Find Inverse function of  $f(x)$ , which is  $f^{-1}(x)$  where  $f(x-1) = x^2 + 2x$

29. If  $m(x) = x^3 + 1$  and  $n(x) = \sqrt[3]{x-5}$ , find the composition  $(n \circ m)(x)$ .

30. Evaluate the limit  $\lim_{x \rightarrow \infty} k(x)$ , where  $k(x) = (3x-5)^2 - (2x+7)^4$ .

31. Solve the inequality:  $\frac{3(x+2)}{(x-2)(x+4)} \geq 0$ .

32. Determine the equation of the ellipse shown in the graph below, with its center at  $(2, -3)$ , horizontal major axis of length 10, and minor axis of length 6.

33. Evaluate the limit:  $\lim_{x \rightarrow -\infty} 4^{-x} - 1$ .

34. What is the function whose graph is a reflection over the y-axis of the graph of  $h(x) = 2 - 4^x$ ? (Reflection over the y-axis)

35. Which of the following functions does not have an inverse function on the specified domain?

(A)  $y = \cos(x)$ , where  $0 \leq x \leq \pi$

(B)  $y = x^2 - 3$

(C)  $y = \frac{1}{x} - 3$

(D)  $y = 3^x$

36. Give an algebraic expression for  $\sin(\cos^{-1}(x))$ . (Algebraic Expression)

37. A circle is graphed using the parametric equations shown below:  $x = 7\sin(t) - 2$  and  $y = 7\cos(t) + 4$  Where is the center of the circle located? (Parametric Equations)

**38. The table shows the predicted growth of a particular bacteria population after various numbers of hours. Write an explicit formula for the sequence of the number of bacteria. (Arithmetic Sequence)**

Hours ( $n$ )	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
Bacteria ( $b_n$ )	23	46	69	92	115

**39. What are the points where the graph of the polynomial  $g(x) = 3(x + 6)(x - 6)^2$  crosses the x-axis? (Polynomial Zeros)**

**40. Evaluate:  $\lim_{x \rightarrow \infty} \frac{2x^2 - 5x - 7}{4x^2 + 5x - 9}$  (Limits)**

**41. Given the function  $g(x) = (x^2 - 4) - (x - 2)(x + 1)$ , at what value of  $x$  is the absolute maximum of  $g(x)$  over the interval  $[-1.5, 3]$ ? (Maximum Value of a Function)**

**42. Approximate  $\log_5 36$ , given that  $\log_2 5 \approx 2.33$  and  $\log_2 3 \approx 1.58$ .**

**43. (Exponential Equation): Solve for  $y$ :  $5^{2y} = 25^{y+1}$ .**

**44. Which of the following choices is equivalent to the complex number  $1 - 4i$ ? (Complex Number Representation)**

**45. Find the angle between two vectors  $\vec{r}$  and  $\vec{s}$  with magnitudes 3 and 4, respectively, that has a dot product equal to 6. (Vector Dot Product)**

**46. How long will it take for 500 to triple in value in an investment when interest is compounded continuously at the rate of 4.2% per annum? Round your answer to the nearest year. (Compound Interest)**

47. (Projectile Motion): A rock is thrown vertically upward from a cliff that is 100 feet above ground level with an initial velocity of 60 feet per second. The height  $h$ , in feet above ground level, of the rock  $t$  seconds after the throw is given by the function  $h(t) = -16t^2 + 60t + 100$ . At approximately what value of  $t$  will the rock be at the height of 80 feet and moving upward?

48. (Exponential Decay): A ball is dropped from a height of 25 feet. After each bounce, the ball reaches 80% of its previous height. How high will the ball rebound after the second bounce?

49. (Exponential Decay): Suppose you release a balloon from a height of 20 feet. After it ascends, it stabilizes at 120% of its previous height each minute. How high, to the nearest tenth, will the balloon be after 2 minutes?

50. What is the third term in the expansion of  $(2a + 3b)^5$ ?

51. Simplify:  $\ln\left(\sqrt[3]{e^3 x}\right)$ .

52. If  $\sec x \neq 1$ , which of the following is equivalent to  $\frac{\tan^2 x}{1 + \sec x}$ ?

53. 4. If  $\cos \alpha = a$ , then what is  $\sin \alpha \cdot \cos \alpha \cdot \cot \alpha$ ?

54. If  $g(\theta) = m \sin \theta + n$ , what is the maximum value of  $g(\theta)$ ?

55. Which of the following equations could represent the graph shown below, assuming the graph depicts a simple transformation of a basic trigonometric function with a period of  $\pi$  and an amplitude of 1?



56. Which of the following expressions is equivalent to  $\sec\left(\frac{3\pi}{4}\right)$ ?

(A)  $y = -\frac{1}{2}\cos(2x)$

(B)  $\tan\frac{3\pi}{4}$

(C)  $\csc\left(-\frac{3\pi}{4}\right)$

(D)  $\cos\frac{3\pi}{4}$

57. In which quadrant is the terminal side of angle  $\phi$  located if the graphs of  $y = \tan\phi$  and  $y = \sec\phi$  are both increasing when angle  $\phi$  is increasing?

58. Evaluate:  $\sec\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$

59. For the expression  $k - \frac{1}{\cos^2\phi} = \cos^2\phi$  to be an identity, what does k equal?

(A) 1

(B) 0

(C)  $\sin^2\phi$

(D)  $\tan^2\phi$

60. What is the expression  $\frac{\cos 2\phi}{2\sin\phi}$  equivalent to?

61. In the interval  $0 \leq x < 2\pi$ , what are the solutions of the equation  $\cos^2 x = \cos x$ ?

62. What is the expression  $\frac{\cos(x - \frac{\pi}{2})}{\sin x}$  equivalent to?

63. If  $\cos C = \frac{4}{5}$  and  $\cos D = \frac{5}{13}$ , and if C and D are acute angles, what is the value of  $\sin(C + D)$ ?

64. Which of the following choices represents the graph of  $r = 4 \sin \theta$  in polar coordinates?

- (A) A circle centered at the pole with a radius of 4.
- (B) A cardioid that starts at the pole and extends to the right.
- (C) A limaçon with an inner loop.
- (D) A circle centered on the horizontal axis, 2 units to the right of the pole.

65. Which of the following points does not change the location of the point  $(3, \frac{3\pi}{4})$  in polar coordinates?

- (A)  $(3, \frac{11\pi}{4})$
- (B)  $(-3, \frac{7\pi}{4})$
- (C)  $(-3, \frac{3\pi}{4})$
- (D)  $(3, \frac{-5\pi}{4})$

66. Given the polar coordinates  $(5, -\frac{\pi}{6})$ , find the rectangular coordinates of this point.

67. Given the rectangular coordinates (2, -2), find the polar coordinates of this point.

68. If  $h(x) = x^2 - 4x + 3$  and  $p(x) = 3 - x$ , then what does  $p(h(x))$  equal?

(A)  $3x^2 - 12x + 6$

(B)  $x^2 - 4x$

(C)  $-x^2 + 4x$

(D)  $9 - x^2 + 4x - 3$

69. Let  $g$  be a function defined for all real numbers. Which of the following conditions is not sufficient to guarantee that  $g$  has an inverse function?

(A)  $g$  is one-to-one.

(B)  $g$  has a continuous, non-repeating range.

(C)  $g$  has no critical points.

(D)  $g$  passes the Horizontal Line Test in its domain.

70. Which of the following functions is not even?

(A)  $q(x) = \cos(x)$

(B)  $r(x) = \cos(3x)$

(C)  $s(x) = x^6$

(D)  $t(x) = \frac{x^3}{x^2 + 1}$

71. At what value(s) of  $x$  do the graphs of  $y = 3x + 1$  and  $y^2 = 9 - x^2$  intersect?

72. If  $g^{-1}(x)$  is the inverse of  $g(x) = 10^x$ , then what does  $g^{-1}(x)$  equal?

73. Find  $g(x+k)$  when  $g(x) = 3x^2 + 5x + 2$ .

74. Given the Markov matrix below, which of the following statements is true about the variable  $b$  ?

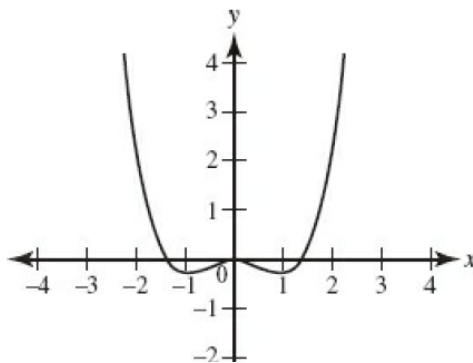
$$\begin{bmatrix} b & 1-b & 0 \\ 0 & b & 1-b \\ 1-b & 0 & b \end{bmatrix}$$

- (A)  $b$  is any real number
- (B)  $b$  is any positive real number
- (C)  $0 \leq b \leq 1$
- (D)  $b = 1$

75. For  $G(x) = g(x) + 2$  where  $g(x) = \begin{cases} 9x - 2, & \text{if } x < 3 \\ 5 - x, & \text{if } x \geq 3 \end{cases}$ , evaluate  $G(7)$ .

- (A) 62
- (B) -2
- (C) 0
- (D) 5

76. Use the graph of  $f(x)$  below to determine the intervals where  $f(x)$  is increasing and where  $f(x)$  is decreasing. Find correct one.



- (A) Increasing:  $(-2,1)$ ; decreasing:  $(-\infty,-2) \cup (1,\infty)$   
 (B) Increasing:  $(-2,0) \cup (1,\infty)$ ; decreasing:  $(-\infty,-2) \cup (0,1)$   
 (C) Increasing:  $(-1,\infty)$ ; decreasing:  $(-\infty,-1)$   
 (D) Increasing:  $(-\infty,-2) \cup (1,\infty)$ ; decreasing:  $(-2,1)$

77. Using the tables below, evaluate  $(h \circ f)(7)$ . The tables provide values for functions  $f$  and  $h$  for various inputs.

<b>x</b>	<b>2</b>	<b>7</b>	<b>9</b>	<b>11</b>
<b>f(x)</b>	<b>5</b>	<b>3</b>	<b>8</b>	<b>6</b>

<b>x</b>	<b>3</b>	<b>5</b>	<b>8</b>	<b>6</b>
<b>h(x)</b>	<b>12</b>	<b>14</b>	<b>7</b>	<b>15</b>

- (A) 12  
 (B) 14  
 (C) 7  
 (D) 15

78. Find the matrix product  $CD$  if it is defined, given that  $C = \begin{bmatrix} 2 & -1 \\ 4 & 6 \end{bmatrix}$  and  $D = \begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix}$ .

79. Calculate the area of the triangle with the vertices  $(1,1)$ ,  $(4,3)$ , and  $(6,7)$ .

80. Find the vertical asymptote, if any, for the rational function  $h(x) = \frac{3x^2 - 2x - 1}{4x - 1}$ .

81. A garden planning app allocates a certain number of pixels on screen to represent lengths in a garden layout. If a user has a budget that allows for 600 pixels of length to design their rectangular garden, express the area  $A$  of the garden as a function of the width  $w$ , in pixels, of the rectangle.

82. If  $x = 3$  is a real zero of the polynomial  $g(x) = x^3 - 9x^2 + 27x - 27$ , write  $g(x)$  as a product of linear factors.

83. Find the inverse of the matrix  $B = \begin{bmatrix} 2 & -3 \\ 0 & 6 \end{bmatrix}$  if it exists.

84. A deposit of 15,000 is made in an account that earns 4.5% interest compounded monthly. The balance in the account after  $m$  months is given by the sequence  $b_m = 15,000 \left( 1 + \frac{0.045}{12} \right)^m$ . Find the balance in the account after 3 years.

85. If  $\log_5(x+2) - \log_5(x-2)$ , then  $x$  lies in which of the following intervals?

(A)  $(2, \infty)$

(B)  $(0, 2)$

(C)  $(-\infty, -2)$

(D)  $(-2, 0)$

86. If  $\theta$  is an angle in standard position and its terminal side passes through the point  $P(-0.5, 0.5)$  on the unit circle, which of the following is a possible value radian value of  $\theta$  to the nearest hundredth?

- (A) 2.36
- (B) 3.14
- (C) 4.71
- (D) 5.50

87. A company finds that the revenue  $R$ , in dollars, from selling  $q$  units of a product is given by the revenue function  $R(q) = 3q^2 - 18q + 500$ . Determine the number of units sold that will maximize the revenue.

88. An object is thrown vertically upward, and its height  $t$  seconds after it is thrown is given by the equation  $H(t) = 5 + 32t - 16t^2$ . Calculate the average velocity of the object over the interval from  $t = 1$  to  $t = 3$  seconds.

89. Solve the inequality  $\frac{3x+1}{2x+3} > 4$ .

90. Given the values of a function  $g(y)$  at various points, determine between which consecutive values of  $y$  does  $g(y)$  change sign, thus indicating the presence of a root.

<b>y</b>	-2	<b>0</b>	<b>2</b>	<b>4</b>
<b>g(y)</b>	7.14	-12.32	-0.48	13.92

91. Determine the domain of the function  $g(x) = \sqrt{m-x} - \frac{1}{\sqrt{n-x}}$ , where  $0 < m < n$ ,  $0 < x < n$ .

92. Calculate the range of values for  $y$  for which the equation  $3^{y^2} + 3^y - 4 = 0$  has real solutions, rounding your answer to 2 decimal places.

93. If the pattern of growth for a certain species of bacteria doubles every hour, and there are 500 bacteria at time 0, estimate the population of the bacteria after 6 hours.

94. A box with a square base and open top is to be constructed from a square piece of cardboard with sides of 24 inches by removing equal squares of side  $y$  at each corner and folding up the flaps. What should be the side length of the squares cut from each corner to maximize the volume of the box?

95. Let  $T_3 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a reflection about the  $y$ -axis, and let  $T_4 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a dilation with factor  $k = -2$ . Find the standard matrix for the composition  $T_4 \circ T_3$  on  $\mathbb{R}^2$ .

96. A linear function  $Q$  is used to model the quantity, in thousands, of a certain product sold as a function of its price  $x$ , in dollars. It is known that  $Q(10) = 40$  and  $Q(15) = 25$ . Based on this model, which of the following is true?

A) For each dollar increase in price, the quantity sold increases by approximately 3000 units.

B) For each dollar increase in price, the quantity sold decreases by approximately 1500 units.

C) For each dollar increase in price, the quantity sold decreases by approximately 3000 units.

D) For each dollar increase in price, the quantity sold increases by approximately 1500 units.



97. The function  $g$  is given by  $g(z) = z^4 - z^2$ . Which of the following statements is true and supports the claim that  $g$  is an even function and not an odd function?

A)  $g(0) = g(-0)$

B)  $g(2) = g(-2)$

C)  $-g(2) = g(-2)$

D)  $g(2) = -g(-2)$

98. The polynomial function  $m$  is given by  $m(x) = x^4 - 6x^3 + 8$ . Which of the following describes the behavior of  $m$  as the input values increase without bound?

A) The output values decrease without bound.

B) The output values increase without bound.

C) The output values sometimes increase without bound and sometimes decrease without bound.

D) The output values get closer to 8.

99. The table gives values for the function  $g$  at selected values of  $x$ . Which of the following conclusions with reason is consistent with the values in the table?

$x$	1	2	3	4	6
$g(x)$	3	8	15	24	35

A) The graph of  $g$  is concave up because the second differences of  $g(x)$  are constant and positive.

B) The graph of  $g$  is concave down because the second differences of  $g(x)$  are constant and positive.

C) The graph of  $g$  is linear because the first differences of  $g(x)$  are constant.

D) The graph of  $g$  is concave up because the first differences of  $g(x)$  are increasing.

**100. The polynomial function  $h$  is given by  $h(x) = cx^d$ , where  $c$  is a non-zero constant and  $d$  is a positive integer. It is known that  $\lim_{x \rightarrow \infty} h(x) = \infty$  and  $\lim_{x \rightarrow -\infty} h(x) = -\infty$ . Which of the following statements must be true?**

- A) The value of  $c$  must be positive, because as  $x$  increases without bound or decreases without bound, the end behaviors of  $h$  are different.
- B) The value of  $c$  must be negative, because as  $x$  increases without bound or decreases without bound, the end behaviors of  $h$  are the same.
- C) The value of  $d$  must be even, because as  $x$  increases without bound or decreases without bound, the end behaviors of  $h$  are the same.
- D) The value of  $d$  must be odd, because as  $x$  increases without bound or decreases without bound, the end behaviors of  $h$  are different.

**101. The function  $L$  models the level of a certain medication in the bloodstream and is given by  $L(t) = \frac{300t}{1+3t}$  for  $t \geq 0$ , where  $t$  is measured in hours since the medication was taken. Which of the following describes the level of the medication in the bloodstream as time increases?**

- A) The level decreases and approaches a value of 0 mg/L.
- B) The level increases and approaches a value of 100 mg/L.
- C) The level increases and approaches a value of 300 mg/L.
- D) The medication level increases without bound.

**102. The function  $s$  is given by  $s(x) = \frac{x+2}{x-4}$ . What are all solutions to  $s(x) < 0$ ?**

- A)  $x < -2$  and  $x > 4$
- B)  $x < 4$  only
- C)  $x > -2$  only
- D)  $x > -2$  and  $x < 4$

**103. The zeros of a rational function  $h$  are 2 and -3. Which of the following expressions could define  $h(x)$  ?**

A)  $\frac{(x-1)(x+4)}{(x-2)(x+3)}$

B)  $\frac{(x-2)(x+3)}{(x-1)(x+4)}$

C)  $\frac{(x-2)(x+3)}{(x+2)(x-3)}$

D)  $\frac{(x+2)(x-3)}{(x-2)(x+3)}$

**104. For the function  $h$  , it is known that  $h(2) = 0$  and  $h(5) = -3$  . The function  $k$  is given by  $k(x) = h(x-3)$  . Which of the following must be a solution to  $k(x) = 0$  ?**

A)  $x = -1$

B)  $x = 1$

C)  $x = 5$

D)  $x = 8$

**105. Which of the following functions has the same end behavior as the rational function  $q$**

**given by  $q(x) = \frac{3x^2 + 4x - 7}{5x^2 - x + 2}$  ?**

A)  $f(x) = \frac{3}{5}$

B)  $g(x) = 1$

C)  $h(x) = -\frac{3}{5}$

D)  $k(x) = x$

**106.** The rational function  $m$  is given by  $m(x) = \frac{(x-1)^2(x+3)(x-4)}{x^2(x-1)(x+2)^3}$ . For which of the following values of  $x$  does the graph of  $m$  have vertical asymptotes?

- A)  $x=1$  and  $x=-2$  only
- B)  $x=0$  and  $x=-2$  only
- C)  $x=0$ ,  $x=1$ , and  $x=-2$
- D)  $x=0$ ,  $x=1$ , and  $x=4$

**107.** The domain of the function  $f$  is  $-4 \leq x \leq 16$ . If the function  $k$  is given by  $k(x) = 2\left(\frac{x}{3}\right) + 1$ , what is the domain of  $k$ ?

**108.** The domain of the function  $f(x)$  is  $-4 \leq x \leq 16$ . If the function  $k(x)$  is given by

$k(x) = f\left(\frac{x}{3}\right) + 1$ , what is the domain of  $k$ ?